Towards a statistical physics analysis of on-line learning in deep ReLU neural networks

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Techniques from statistical physics can be used in the analysis of machine learning algorithms. Machine learning models, and in particular neural networks, consist of a large number of adaptive weights. Under special assumptions, it becomes possible to model the macroscopic learning behavior of these systems by a set of deterministic differential equations. Examples of the approach for the analysis of on-line learning in two-layer sigmoidal neural networks can be found in [1, 2]. Recently, a first statistical physics analysis of on-line gradient descent learning in two-layer ReLU neural networks has been done [3]. Now, the aim is to analyze within the framework the learning behavior of more extended architectures: First, the previously studied two-layer ReLU network will be augmented with biases and second layer weights. This gives rise to a machine that is capable of representing any real-valued continuous function on compact subsets of $\mathbb{R}^N$, a so-called *universal approximator*, see [4, 5], and proved specifically for ReLU activation in [6]. Secondly, we will revisit tree-like architectures, in which the neurons' receptive fields are non-overlapping. The consideration of these tree-like networks may prove as an important step in a potential extension of the theory towards deep neural networks.

References

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